

Chapter 4. Basic Tools and Techniques

As discussed, the project is based on mental physics which in turn is the application of Kantian metaphysics to mind-brain. The approach follows Bacon's investigative method (Mental Physics, pp.41) and employs set-membership theory (SMT) [Combettes, 1992]. The basic essence of SMT is that a problem may have more than one mathematical solution (> 1 satisfactory point) such that any one of them when picked makes no practical difference. The Slepian two-world model of facet-A and facet-B for principal quantities discussed earlier is an example of SMT.

In addition to these the tools for constructing the neural network are embedding field theory and method of minimal anatomies. They are both interlinked techniques and are discussed separately below.

Set-Membership Theory (SMT)

SMT produces a solution set whose characteristic is consistent with observed data and available a priori knowledge. The estimate made by SMT is governed by the notion of feasibility.

Figure 4.1 illustrates SMT. The figure (Fig.4.1a) shows the mathematical description, also called set-theoretic estimation. It is based on the point-estimation problem of guessing the value of an object h belonging in a solution space Ξ for some given observed data and some a priori knowledge. The figure (Fig.4.1b) also shows a simple practical application of the mathematics.

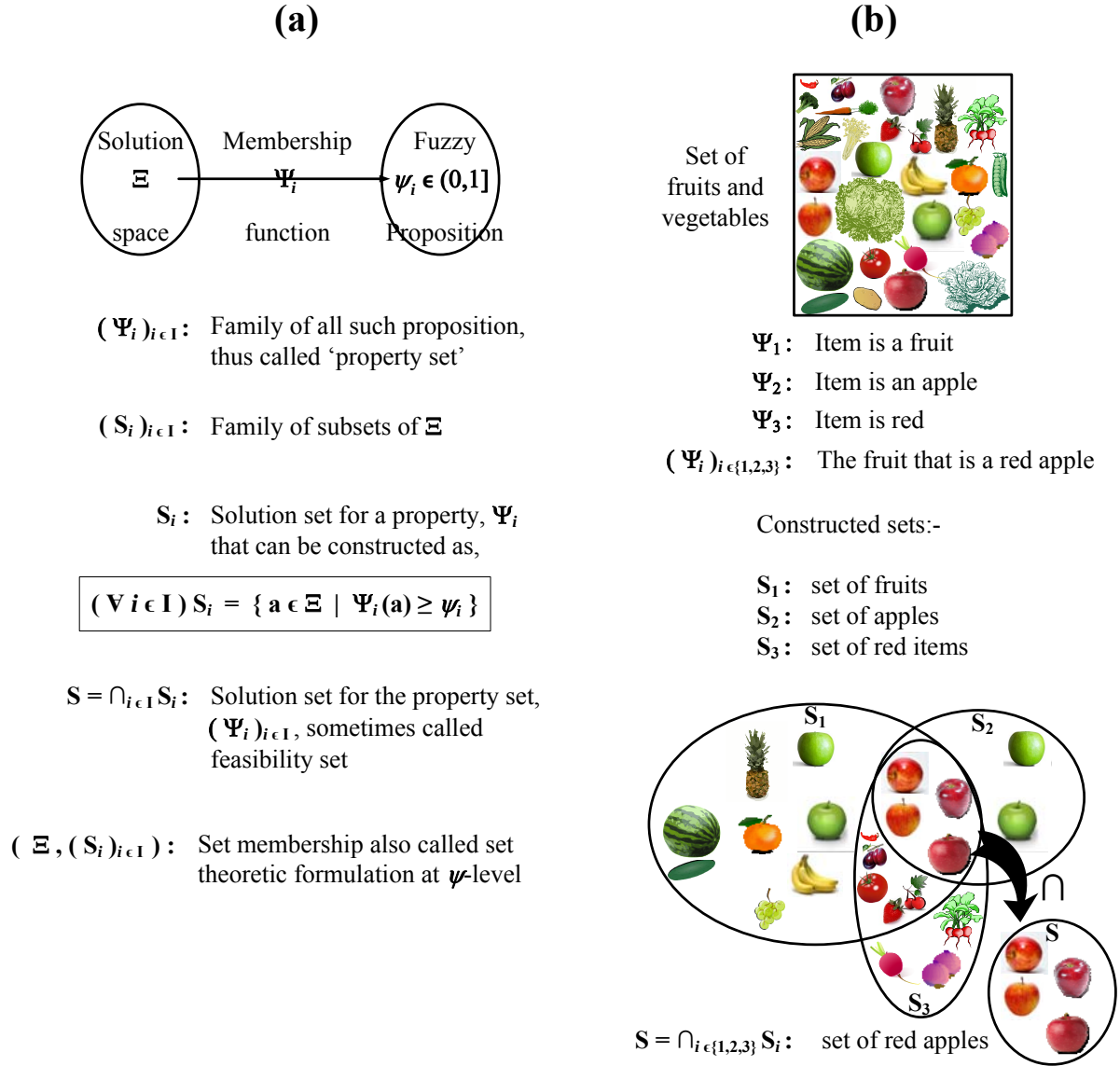


Figure 4.1. Illustration of Set-Membership Theory (SMT). (a) Mathematical description for constructing the feasibility set (solution set for the property set). The membership function or property Ψ_i is a proposition which determines the contents of its solution set S_i such that the contents are consistent with the information carried by Ψ_i at ψ -level. For crisp non-fuzzy propositions, $\psi_i \in \{0, 1\}$. (b) Given a set of fruits and vegetable and a priori knowledge that the item from the set is a fruit (Ψ_1), an apple (Ψ_2) and red (Ψ_3) in color, solution sets (S_1 , S_2 , S_3) for respective properties are constructed. This is followed by the construction of the feasibility set (S) which is the solution set for the property set $(\Psi_i)_{i \in I}$ that the ‘fruit is a red apple’.

There are at least two main steps for posing the set membership, $(\Xi, (S_i)_{i \in I})$. They are:

1) construction of property sets, $(\Psi_i)_{i \in I}$, and 2) selection of solution space, Ξ . These two

steps are connected. There is no general technique to define membership function (a property, Ψ_i) in an objective and systematic way. In this project, property set is constructed such that it is epistemologically correct.

An important criteria of selecting the solution space is that it must be able to model available information easily and accurately. But not all information are available in most cases. Thus, the selected solution space should contain objects that are directly described from available information and reformulate the rest of the information in this space. The solution space and hence its resulting solution sets S_i should be compatible with the feasibility set, S (solution set for the property set).

The selected solution space therefore implicitly models a priori information. The constraints imposed by the selected solution space and feasibility set incorporates all available properties (information) about the problem.

Embedding Field Theory (EFT)

Theory of embedding field developed by Stephen Grossberg [Grossberg, 1969b, 1971] is a general theory of neural networks. This is derived systematically as follows. Simple psychological postulates (familiar behavioral facts) are used to derive neural networks which are then analyzed for psychological consequences. Psychological predictions are then used to derive a learning theory within the theory of embedding fields (EFT).

Mathematically, this means that EFT is the neural network or machine described by cross-correlated flows on signed (+ or excitatory, – or inhibitory) directed networks. The flows obey systems of non-linear differential equations (NLDE). Thus, deriving EFT from simple psychological postulates means derivation of NLDE.

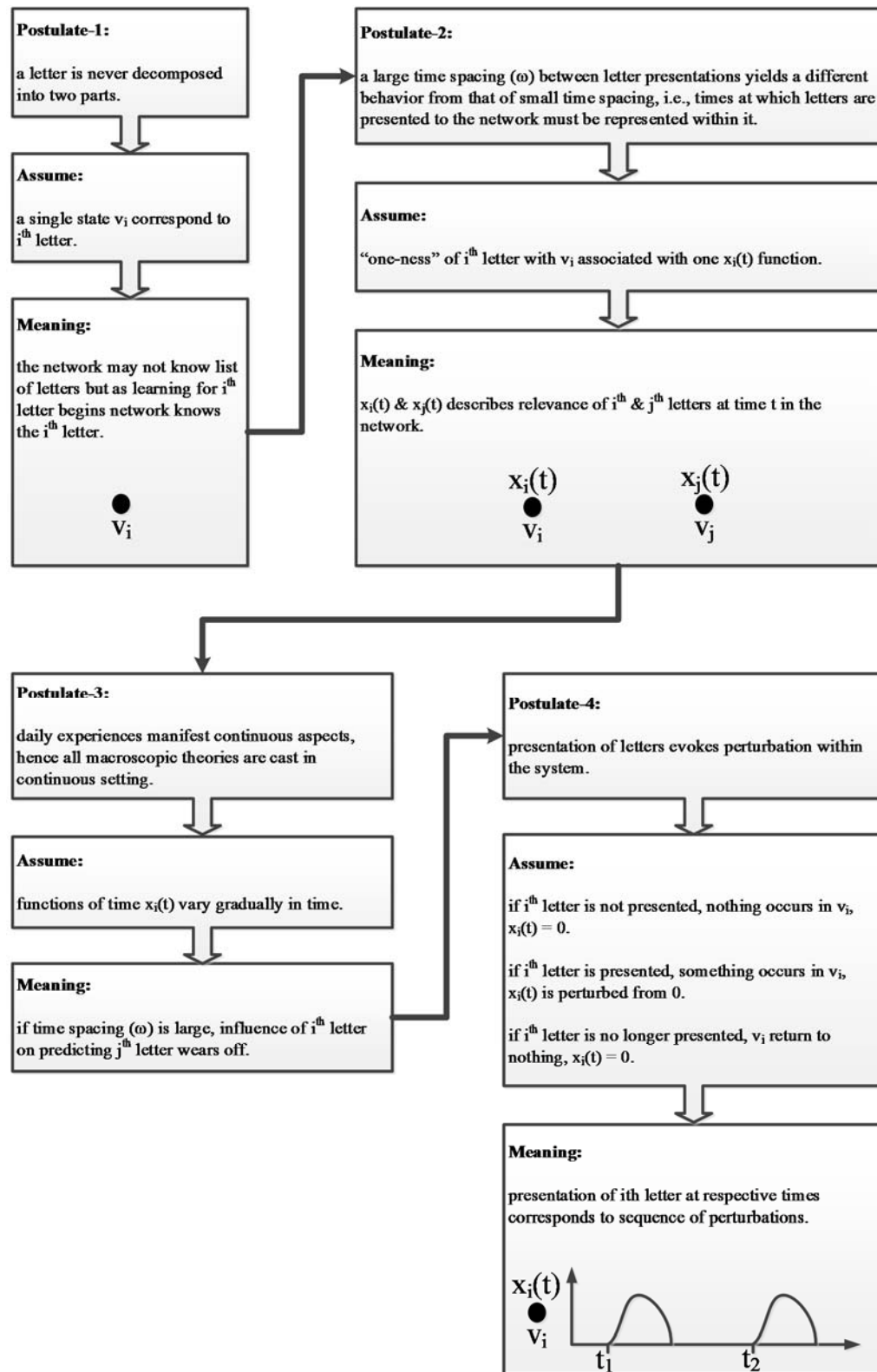


Figure 4.2. Postulates 1 to 4 and resulting derivations for the simple network. With each additional postulate the derived model acquired more capabilities. The meaning implications noted in this figure have objectively valid expression in mental physics.

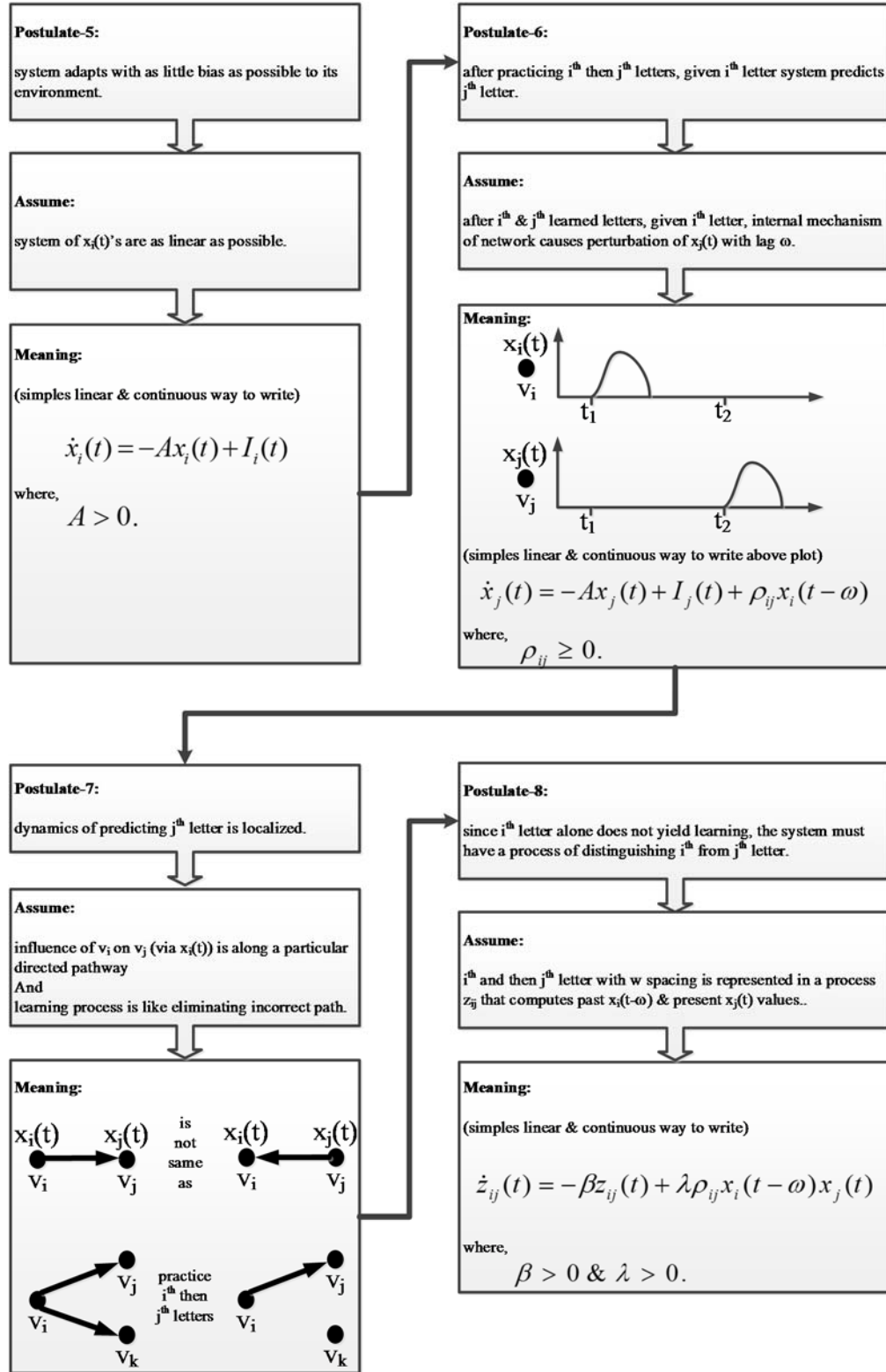


Figure 4.3. Postulates 5 to 8 and resulting derivations for the simple network. Note that this is a continuation of figure 4.2.

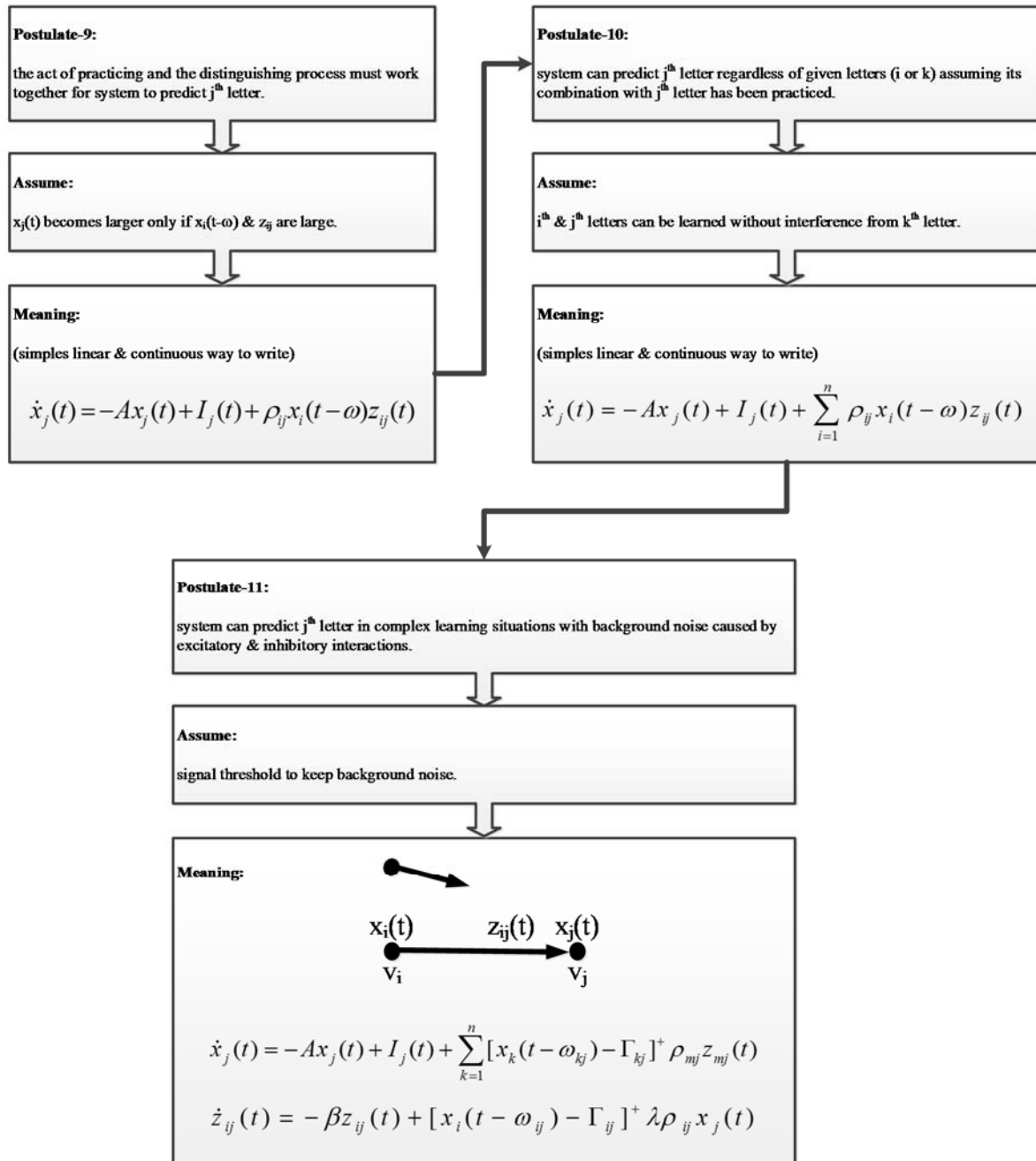


Figure 4.4. Postulates 9 to 11 and resulting derivations for the simple network. This is a continuation of figure 4.3. The non-linear differential equation (NLDE) of the finally derived simple network is the two equations resulting after postulate-11.

Figures 4.2, 4.3 and 4.4 show a step-by-step derivation of a simple network with the capability to learn and therefore exhibit learned behaviors. The specific example illustrated is Grossberg's "letter recognition" problem.

Note that though simple behavioral facts and hence empirical terminology are used to derive the network (NLDE), the NLDE variables are secondary quantities. Thus mathematical variables are not necessarily connected to empirical interpretations. However, if the output variable is defined as representing principal quantities empirical connections can be made.

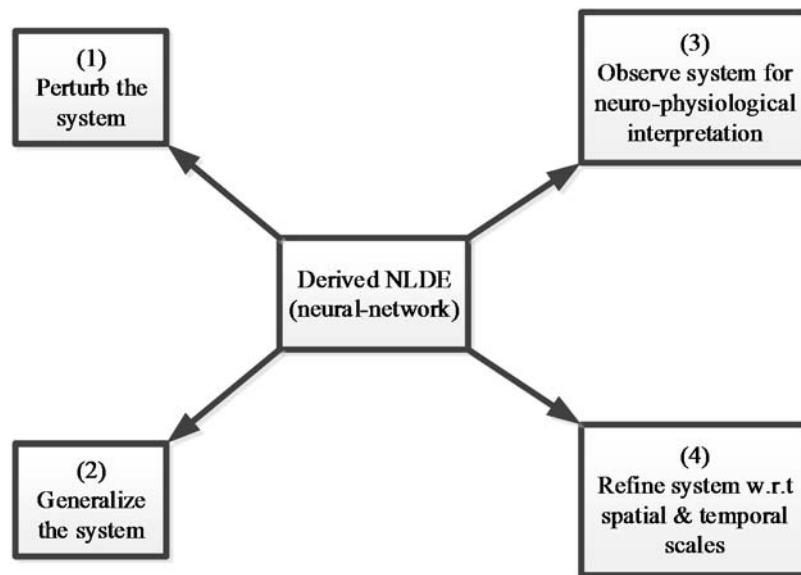


Figure 4.5. Four possible paths after deriving non-linear differential equation (NLDE).

After deriving the derived NLDE (i.e., neural network) there are at least four possible paths for analyzing it (Fig.4.5). Let us we consider system (neural network) input and output as representation of system experience and perturbation cases as representation of mathematical behavior which is qualitatively same as psychological interpretation. Then perturbing the system (Fig.4.5(1)) complicates experiences and provokes various perturbation cases.

The system may also be generalized (Fig.4.5(2)). This implies associating the system to farthest possible psychological experience. Thus, to generalize the system is to investigate the system as pure mathematical problem.

Another path is to observe system for neurophysiological interpretation (Fig.4.5(3)). However one must be careful taking this path and not make leaping interpretations because the system is derived from psychological experiences and does not incorporate microscopic neurophysiology.

Finally, the system may be refined with respect to spatial and temporal scales of the NLDE (Fig.4.5(4)). According to Grossberg, using this approach the system can make hypothesis on possible transients [Grossberg, 1971]. The discussed approaches are summarized in figure 4.6.

In this project, the derived system will be perturbed. This is because qualitative interpretation of the math and hence the psychology of complex (unrelated and related) cases are manifestations of the simple behavioral facts used for the derivation.

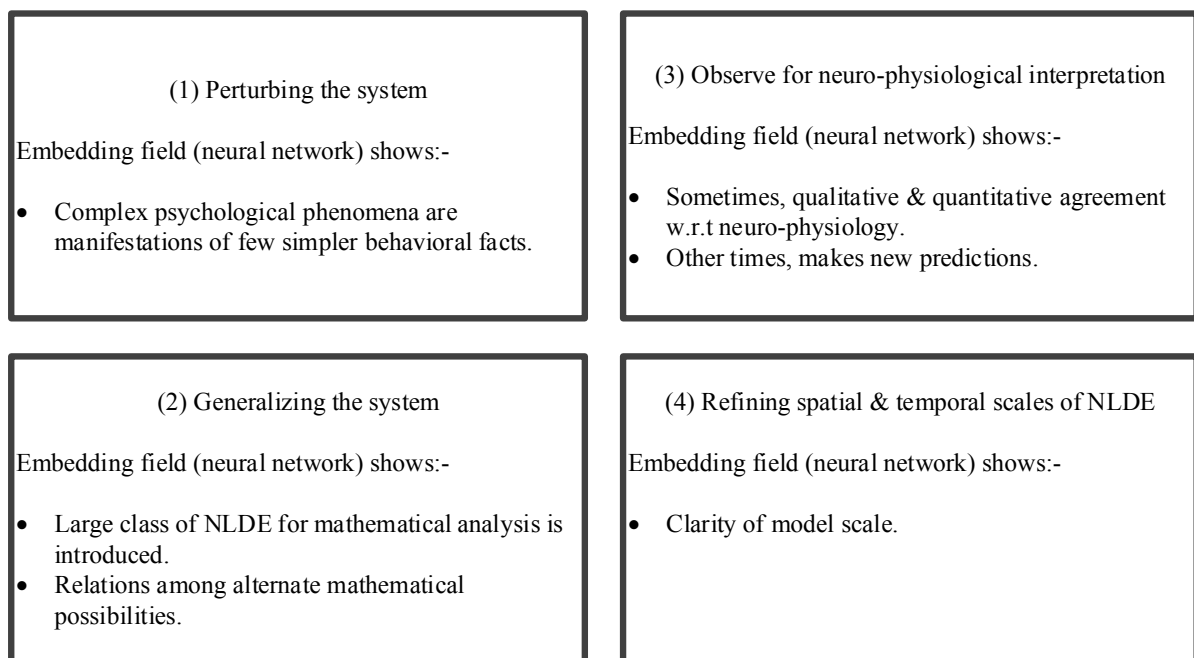


Figure 4.6. Accomplishments of embedding field theory (EFT) depending on the paths taken (Fig.4.4).

Method of Minimal Anatomies (MMA)

Stephen Grossberg introduced MMA as a method of successive approximations [Grossberg, 1982]. Its principle thus has analogies in other scientific fields such as physics, (1-body to 2-body to 3-body and so on) or from thermodynamics to statistical mechanics. Thus with each successive analysis the complexity increases and is more realistic. This is consistent with Bacon's investigative method.

Grossberg's MMA answers the question of 'How to approach several levels of description relevant to understanding behavior?' The embedding field type neural component is first derived from psychological postulates (Fig.4.7). This then forms the neural unit when interconnected to form a specific anatomy. The specific anatomy or system receives input with psychological interpretation resulting in psychologically interpretable output.

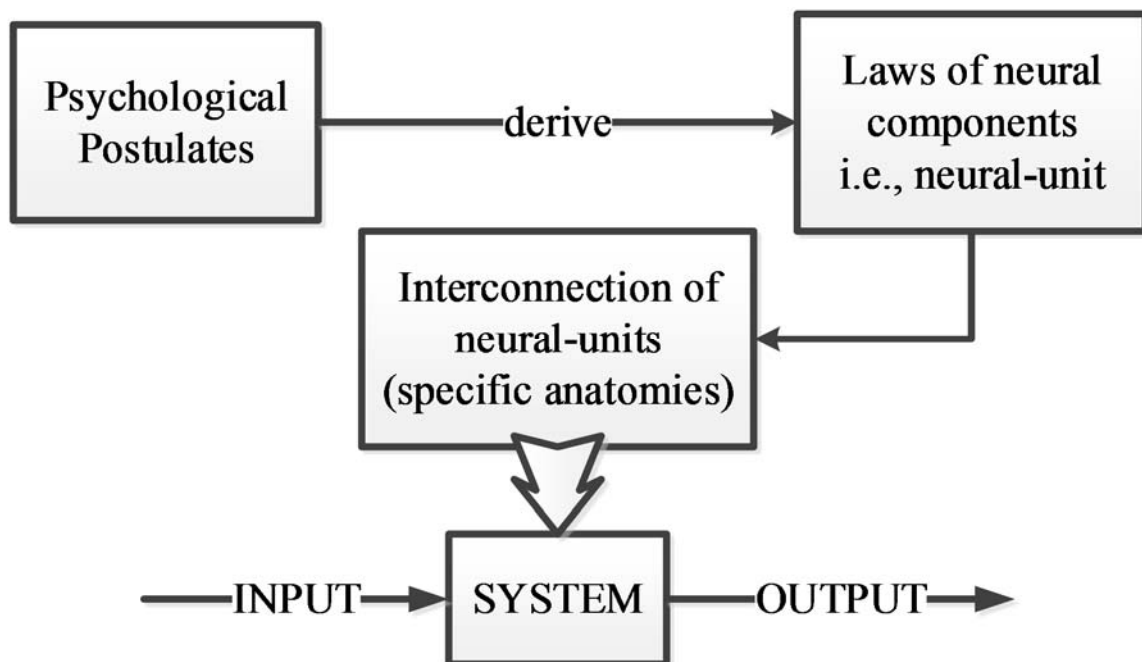


Figure 4.7. Basic description of applying embedding field theory (EFT) to derive a neural network.

It should be highlighted that though the derived components are called neural-units they are not the same as a biologist's understanding of neural components. This is because the derivation is from psychological postulates and hence modelling scale (neural network) is much larger than individual neurons. Also, regardless of how precise the knowledge of laws and postulates may be, accounting for every (10^{12} nerve cells) connections and analyzing them would be impractical [Grossberg, 1982].

The above description (Fig.4.7) is basically an application of MMA. A model (network) may be considered the minimal network. For instance the derived network in figure 4.7 can be a minimal network. Other steps will build upon this minimal network resulting in a network which may then be considered minimal network for a larger model continuing this cycle to get an even larger model.

The general MMA approach may be broken down to four basic steps (Fig.4.8). First a model is picked for considering the minimal network. This is then followed by analysis-step (Step 2) which helps in understanding the mechanism of a particular 'minimal' anatomy. In addition, at this step advantages and disadvantages of its variations can also be imagined further enhancing the understanding.

With every additional postulate (Step 4) the corresponding network will be the previous 'minimal' network with added components for the additional capabilities. Thus, the resultant network will be a hierarchy of networks with sophisticated postulates. The mechanisms of each successive 'minimal' network means that the resultant network will have a catalog of mechanisms for use in various situations.

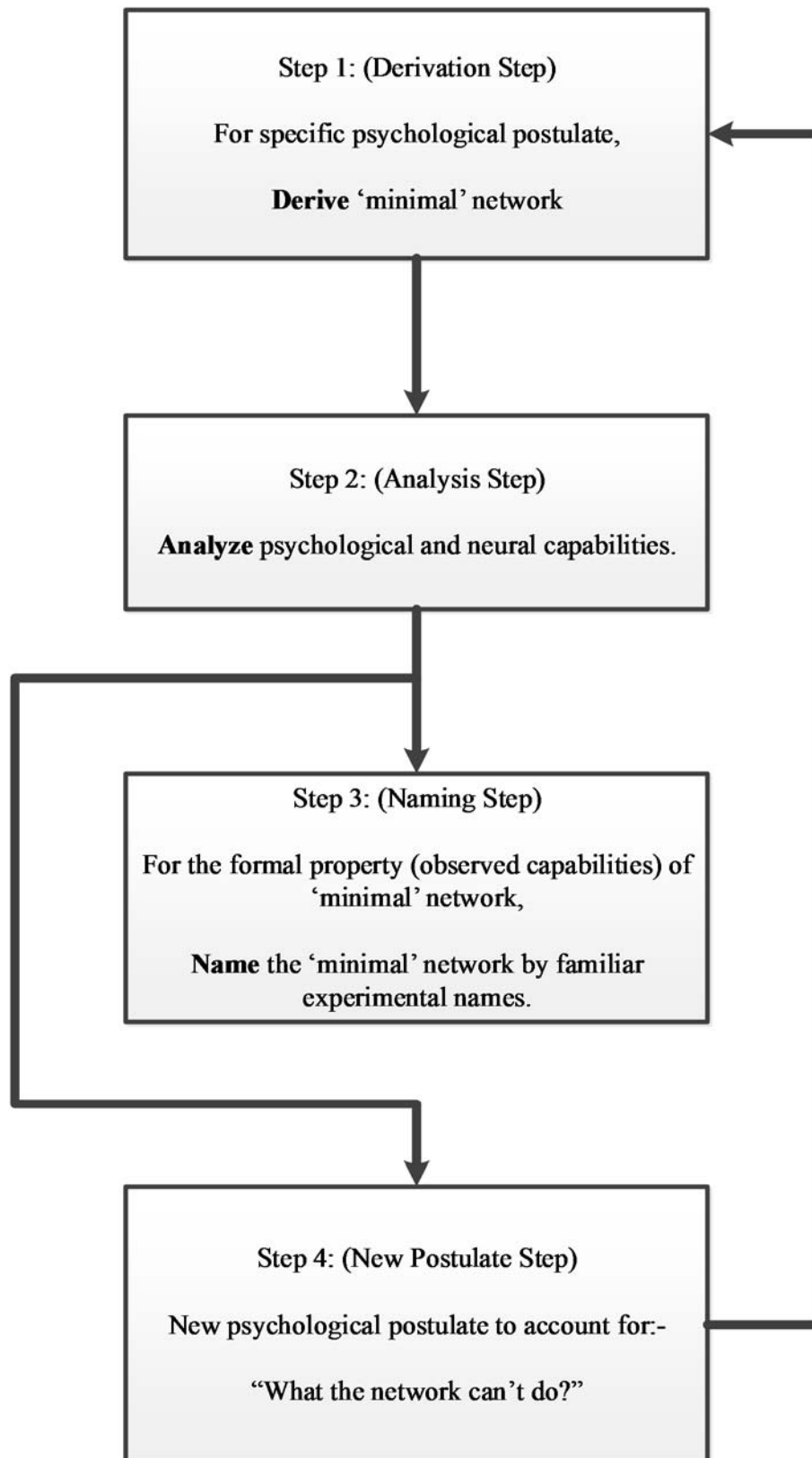


Figure 4.8. A scheme of the method of minimal anatomies approach (MMA) to design a neural network with embedding field type 'minimal' networks.

The naming-step (Step 3) answers the question of “When does a particular phenomenon first appear?” Therefore, this step, regardless of the name, will not compromise the formal correctness of the overall theory from the resultant network. The naming-step is a very useful tool for deeper insight into the resultant neural network.

Finally, one must be wary and to distinguish between postulates, mathematical properties, factual data and interpretation of network variables.

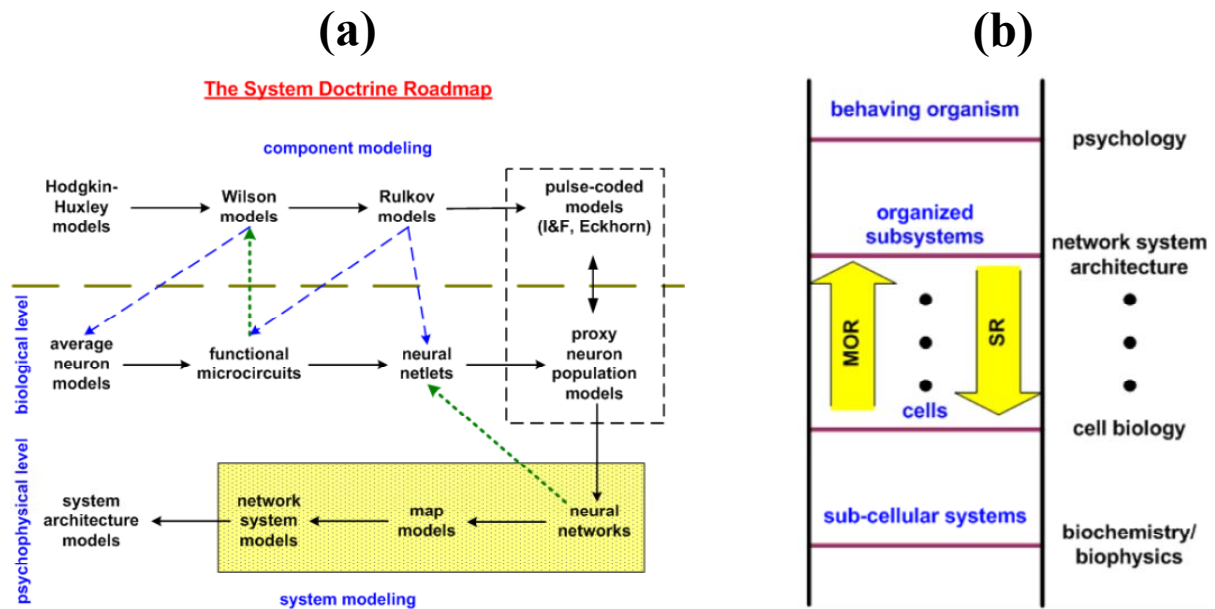


Figure 4.9. Different but similarly meaning description for neuroscience as an interdisciplinary science.

Figure (a) is the neuroscience roadmap view linking biology and psychology. The highlighted region indicates the location of this thesis with regards to the neuroscience roadmap. A map model is a network of neural networks while a network of maps comprises a network system [Wells, Ch.7, 2010].

Figure (b) is another view depicting the neuroscience ladder adopted from [Wells, 2011a] showing several rungs each representing scientific construct at various levels. Moving upward towards increasing level of abstraction from mechanism to behavior is model-order reduction (MOR). However, migration of scientific study from level of observable phenomenon down to increasingly refined scientific constructs is scientific reduction (SR).

Modelling Scale

The above description of MMA involved using EFT to build the network. However, the general idea of MMA may be used to build networks at smaller scales such as pulse-coded neural networks [Sharma, 2011]. Modelling scales for neural networks will be briefly discussed.

If we consider respective disciplines as rungs of a ladder (Fig.4.9), the objective of a neuroscientist should be to join the rungs for achieving the common goal of understanding how mind-brain works. Taking a concept from systems theory, this can be done on two accounts: model-order reduction (MOR) and scientific reduction (SR). MOR is the simplification of the amount of detail needed in obtaining computationally tractable models of ever more complex systems. Thus with MOR one moves towards increasing level of abstraction from mechanism to behavior. SR on the other hand is migrating scientific study and theory from one level of phenomena to others more directly observable by our senses but which use increasingly refined scientific constructs.

For the thesis, the modelling direction will be SR using MMA to build embedding field type minimal-anatomies neural networks to construct a map models.